VISCOUS PROPERTIES OF SOIL

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Dynamic problems connected with the wave propagation in soils not saturated with water and with wave interaction with obstacles and structural elements at the present time are solved on the basis of models in which plastic but not viscous soil properties are taken into account [1-5]. An analysis of experimental data and their comparison with the calculated results [4, 5] confirms that it is permissible to apply the model of an elastic-plastic medium to soils in problems concerning the interaction of waves and structures. At the same time plane-wave damping in soils takes place more intensively than would follow from calculations carried out on the basis of models of an elastic-plastic medium. For example, if in a section of a poured sandy soil, taken as the initial section, the maximum stress in the wave is σ_m=11 kgf/cm^2 and its duration is θ=8 msec, then at a distance of 25 cm the calculations give σ_m=9.5 kgf/cm^2, while the experiment gives σ_m=5 kgf/cm^2. If in the initial section σ_m=20 kgf/cm^2 and θ=6 msec, then at a distance of 35 cm the calculation gives σ_m=17 kgf/cm^2, while the experiment gives σ_m=9 kgf/cm^2. In the calculations it was assumed that unloading takes place with a constant strain. This deviation of the calculated results from the experiment can be explained, in the first place, by the dependence of the $\sigma(\varepsilon)$ on the strain rate $\mathring{\varepsilon}$, which is not taken into account in the model of an elastic-plastic medium. The viscous properties cause additional energy losses and a more intensive damping of the waves. Experimentally the dependence of the σ (ε) curves on the strain rate has been investigated for many soils [5-8]. The dynamic load on the test sample was produced by a body falling from a height or being accelerated by some method. Below we present test results of viscous soil properties when the test sample is compressed by an air shock wave. Compression curves and approximate numerical values of the coefficient of viscosity are obtained.

1. Experimental Determination of Viscous Soil Properties. Usually the dynamic properties are investigated with an impact on the test sample by a body dropped from a height or accelerated by some force. The increase of the load is short-timed, but rapid unloading also follows it. The use of air shock waves of various durations for soil compression allowed us to obtain unloading for a given time.

In the tests compression of medium-grained sand was carried out in a cylinder with smooth walls. The inner diameter of the cylinder was 24 cm. The soil was covered on top by a light aluminum disc 0.2 cm in thickness. In Fig. 1 it is depicted by 1. The air wave moving along the shock tube, in the

TABLE 1

σ_m kgf/cm^2	τ msec	t^* msec	$\varepsilon(\tau)$100	$\varepsilon(t^*)$100	ε+100	$\dot{\varepsilon}$ sec^{-1}
1.8	8	13	1.3	2.0	1.7	1.6
2.7	5	10	2.2	2.7	2.2	2.7
3.9	5	12	2.4	3.5	2.6	2.9
6.0	5	10	4.3	5.6	4.3	5.6

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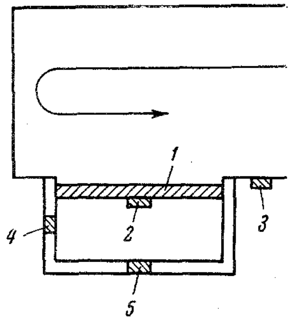

Fig. 1

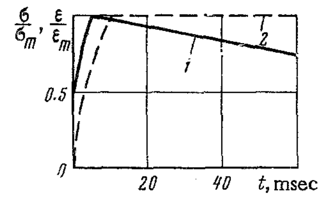

Fig. 2

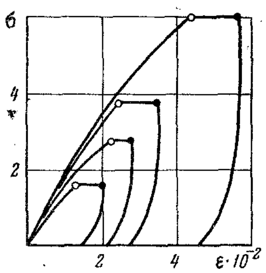

Fig. 3

first-series tests, passed over the disc twice, before and after being reflected from the wall closing the tube. Therefore, the load on the disc reached a maximum after two jumps, approximately after 3 msec. Varying the distance of the cylinder from the wall, we can vary the time of increase of the load on the disc.

In the second series of tests the air wave was reflected directly from the disc; the load increased almost instantaneously. The strain of the soil in both cases was determined from disc displacement recorded against time. The pressure variation in the air wave was determined by the transducer 3, while in the soil it was determined by the transducers 2, 4, and 5.

The tests were carried out for the soil layer thickness h in the cylinder equal to 5 and 9 cm. No differences in the quantities under investigation were observed for different h. In the tests of the first series the volume weight of the soil skeleton was γ=1.55-1.60 g/cm^3; the moisture content was w=3-8%; the velocity of propagation of the maximum stress was c=100 m/sec. In the tests of the second series γ=1.50-1.55 g/cm^3, ω=5-10%, and c=80 m/sec.

In Fig. 2 we have shown the graphs of the relations σ(t) and ε(t), plotted for one of the tests of the first series. In the remaining tests approximately the same behavior of these curves was observed.

Certain average experimental data for the tests of the first series is presented in Table 1.

Here σ_m is the maximum stress in the soil corresponding to the axis of the cylinder, τ is the time of increase of the stress, t* is the time of increase of the strain, ε^+ is the residual strain, $\dot{\varepsilon}$ is the average strain rate for $t \le t^*$.

The duration of the air wave was 200-250 msec.

In Fig. 3 we have shown the experimental graphs of $\sigma(\varepsilon)$ corresponding to four values of maximum stress. The unfilled dots correspond to the maximum stress; the filled dots correspond to the maximum strain. Here, and also in Figs. 4 and 5, the stress in kgf/cm^2 is measured along the ordinate axis. It follows from the graphs that the curves $\sigma(\varepsilon)$ under load are concave relative to the strain axis. At the instant when the maximum stress is reached, the strain is 0.6-0.8 of the maximum. After the maximum stress is reached, creep of the soil is observed for several milliseconds: the strains increase, the stress during this time decreases little, and the relation $\sigma(\varepsilon)$ is practically parallel to the strain axis. During the subsequent fall of the stress to a value approximately equal to 20-40% of the maximum, the strains, reaching the maximum value, practically are unaltered and only afterwards begin to diminish. The residual strains form 0.7-0.9 of the maximum value.

In Figs. 4 and 5 we have represented the graphs of $\sigma(\varepsilon)$ obtained in the tests of the second series, when the load on the soil increased with practically a jump. Fig. 4 corresponds to the primary tests, while Fig. 5 corresponds to secondary tests carried out without restacking the soil after the first test. The pressure in the air wave during the repeated tests was approximately the same as during the corresponding primary tests.

The $\sigma(\varepsilon)$ curves in Figs. 4 and 5 are convex relative to the ε axis. This difference from Fig. 3 is explained by a more rapid increase of the load in the second series of tests. The strain values, in comparison with the graphs of Fig. 3, increase. This is connected with a lower density of the soil. From a comparison of the graphs in Figs. 4 and 5 it follows that for repeated soil compression by an air shock wave the strain is slightly reduced; the corresponding $\sigma(\varepsilon)$ curve approaches the stress axis.

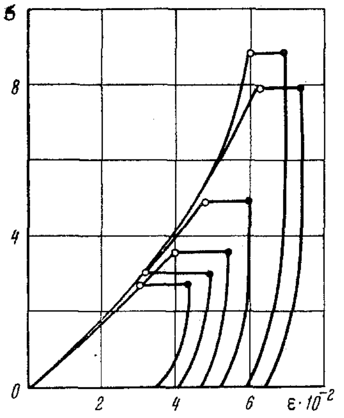

Fig. 4

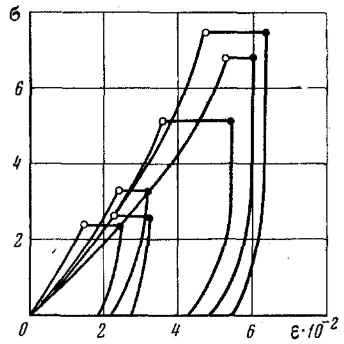

Fig. 5

The unfilled dots in Figs. 4 and 5 determine the curves which in the first approximation corresponds to the primary (Fig. 4) and the secondary (Fig. 5) dynamic (shock) compression of the soil for the strain rate $\dot{\varepsilon} \to \infty$. Through the filled dots we can also draw a curve which can be considered as the limiting (static) pressure drop in the air wave for the conditions being considered. Thus, for repeated compression the difference between the curves of the dynamic and static compression is preserved.

2. Determination of the Coefficient of Viscosity. The existence of a part on the $\sigma(\varepsilon)$ curve where the strain increases while the stress decreases confirms the existence of viscous properties in the soil. In the first approximation, apart from the curves of dynamic and static compression, these properties are also characterized by the coefficient of viscosity η.

To determine the approximate numerical value of η, we compare the experimental values of $\varepsilon(\tau)$, t^*, and $\varepsilon(t^*)$ with calculated values obtained in conformity with the soil model proposed in [5]. The calculations are carried out for various η. The value of η is chosen which corresponds to the highest degree to the experiment.

In conformity with [5], the relation $\sigma(\varepsilon)$ for impact and static compression is taken as linear

$$\sigma = E_1\varepsilon \quad \text{for } \dot{\varepsilon} \to \infty, \quad \sigma = E_2\varepsilon \quad \text{for } \dot{\varepsilon} \to 0. \tag{2.1}$$

The coefficient of viscosity is constant. The elastic limit is zero.

The strain ε of an element of the medium is represented as the sum

$$\varepsilon = \varepsilon_1 + \varepsilon_2.$$

In the period during which the stresses increase these quantities are given by the equations

$$\varepsilon_1 = \frac{\sigma}{E_1}, \qquad \sigma = E_3\varepsilon_2 + \eta\dot{\varepsilon}_2, \qquad \frac{1}{E_3} = \frac{1}{E_2} - \frac{1}{E_1}. \tag{2.2}$$

Then the total strain of an element of the medium is

$$\dot{\varepsilon} + \mu\varepsilon = \frac{\dot{\sigma}}{E_1} + \frac{\mu\sigma}{E_2}, \qquad \mu = \frac{E_3}{\eta}. \tag{2.3}$$

For a stress decrease it is assumed that ε_1 decreases according to a law that is different from (2.1); this gives rise to the residual stresses of the soil

$$\varepsilon_1 = \frac{\sigma_m}{E_1} - \frac{\sigma_m - \sigma}{E^*}, \qquad E^* > E_1, \tag{2.4}$$

where σ_m is the maximum value of the stress thus reached.

TABLE 2

σ_m kgf/cm^2	μ cm^{-1}	t^*msec	$\varepsilon(\tau)100$	$\varepsilon(t^*)100$
	200	22	2.3	3.2
2.7	500	12	2.7	3.2
	1000	8.6	3.0	3.3
6	500	11.7	6.0	7.3

While the stress decreases, ε_2 can either increase or remain constant. If $\varepsilon_2 > 0$, then its variation, as before, is given by the second equation of (2.2). Then the total strain of an element of the medium is

$$\dot{\varepsilon} + \mu\varepsilon = \frac{\dot{\sigma}}{E^*} + \frac{\mu\sigma(E_3 + E^*)}{E_3E^*} + \frac{E^* - E_1}{E_1E^*}\mu\sigma_m. \tag{2.5}$$

For a subsequent decrease of the stress, when ε_2 = const, the total strain is given by the equations

$$\sigma - \sigma^{(1)} = E^*(\varepsilon_1 - \varepsilon_1^{(1)}), \quad \varepsilon = \varepsilon_1 + \varepsilon_2, \tag{2.6}$$

where $\sigma^{(1)}$ and $\varepsilon^{(1)}$ correspond to the maximum ε_2.

Physically, ε_1 expresses the strain caused by the compression of water and salt films located between the grains of soil, and individual protuberances of the grains. This strain includes the reversible (elastic) and irreversible (plastic) parts. The strain ε_2 corresponds to the displacement of the grains, the process of their more compact stacking taking place during a finite time interval. This strain is taken to be irreversible, since in soils not saturated with water and under insufficiently large stresses water and air contained by pores which expand during unloading cannot overcome friction forces between the solid grains to return them to the initial state, as is the case for soils saturated with water. Thus, the model [5] takes into account certain specific properties of the soil and thus differs from other models of viscous media.

In the tests of the first series the stress in the soil increased approximately according to the linear law

$$\sigma = \sigma_m\frac{t}{\tau}, \qquad 0 \leqslant t \leqslant \tau. \tag{2.7}$$

If the law giving the stress increase is known, then (2.3) constitutes the differential equation

$$\dot{\varepsilon} + \mu\varepsilon + Bt + D = 0, \qquad B = -\frac{\mu\sigma_m}{E_2\tau}, \qquad D = -\frac{\sigma_m}{E_1\tau}. \tag{2.8}$$

By integrating, we obtain the dependence of the strain on time in the form

$$\varepsilon(t) = -\frac{D}{\mu} - \frac{B}{\mu^2}(\mu t - 1) + Ne^{-\mu t}, \qquad N = -\frac{B}{\mu^2} + \frac{D}{\mu}. \tag{2.9}$$

For $t \leq \tau$ the stress also decreased approximately according to the linear law

$$\sigma = \sigma_m\frac{\theta - t}{\theta - \tau}. \tag{2.10}$$

Substituting this expression into (2.5), we obtain a differential equation of the form (2.8); integrating this equation, we find the dependence of the strain on time in the form (2.9). Here

$$B = \frac{\mu\sigma_m(E^* + E_3)}{E^*E_3(\theta - \tau)}, \qquad D = \frac{\sigma_m}{E^*(\theta - \tau)} - \frac{\mu\sigma_m(E^* + E_3)\theta}{E_3E^*(\theta - \tau)} - \frac{E^* - E_1}{E_1E^*}\mu\sigma_m,$$

$$N = \left[\varepsilon(\tau) + \frac{D}{\mu} + \frac{B}{\mu^2}(\mu t - 1)\right]e^{\mu\tau}.$$

Knowing the relation $\varepsilon(t)$, we find the time instant t^* when the strain ε reaches the maximum,

$$t^* = -\frac{1}{\mu}\ln\frac{-B}{N\mu^2}. \tag{2.11}$$

In Table 2 we have presented the calculated results of the quantities $\varepsilon(\tau)$, t^*, $\varepsilon(t^*)$; these calculations were carried out according to the equations given above for various values of μ. The density and the velocity of propagation of plastic strains are taken the same as in the soils used to carry out the tests. Here $E^*=4E_1$ and $2E_2=E_1$.

From a comparison of the data presented in Tables 1 and 2 it follows that the experimental and calculated values are most close to one another for $\mu=500$ sec^{-1}. The variation of μ by several tens and even hundreds of units shows, however, small effect on the values of $\varepsilon(\tau)$, t^*, $\varepsilon(t^*)$. Therefore the value $\mu=500$ sec^{-1} is approximate.

Knowing μ, we find the corresponding value of the coefficient of viscosity for the soil under investigation:

in the MKS system

$$\eta=\frac{E_3}{\mu}=3.2.10^4 \text{ kg/msec;}$$

in the CGS system

$$\eta = 3.2 \cdot 10^5 \text{ poise.}$$

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